Indian Statistical Institute, Bangalore
B. Math. Second Year
Second Semester - Computer Science II

Mid-Semester Exam | Duration: 3 hours | Date : March 06, 2015

Answer all the questions. | Max Marks: 30

1. In Octave/Matlab what do **eps, realmin** and **realmax** represent. What is *overflow* and *underflow* ? [5]

2. Let $x \in \mathbb{R}$ and $f(x)$ be a real valued function that is to be evaluated on the computer using Octave/Matlab. As discussed in the class a good model for the representation of a real number and evaluation of a function on the computer is

$$\widetilde{x} = (1 + \epsilon)\ x$$

$$\widetilde{f}(x) = (1 + \mu_f\ \epsilon) f(x)$$

where ϵ and μ_f are the error constants due to the inexact representation of the number x and the function $f(x)$ on the computer. Given $a_0, a_1, ..., a_n \in \mathbb{R}$, consider the sums

$$s_n = \sum_{j=0}^{n} a_j$$

$$\widetilde{s}_n = \sum_{j=0}^{n} \widetilde{a}_j$$

with

$$\widetilde{a}_o = a_0.$$

Calculate the error constant μ_{s_n} for the summation and explain why it is better to use $1/e^x$ to calculate e^{-x} instead of the sum

$$1 - x + \frac{x^2}{2} - \frac{x^3}{3!} \cdots$$

for $x > 0$ and large. [10]

3. Assuming the theorem: Let $\Omega \subset \mathbb{R}$ be closed and $f : \Omega \to \mathbb{R}$ be a continuously differentiable function satisfying $f(\Omega) \subset \Omega$ and for $x, y \in \Omega$

$$|f(x) - f(y)| \leq L\ |x - y| \ \text{ with } \ 0 < L < 1.$$

Then, there exists a unique $z \in \Omega$ satisfying

$$f(z) = z \tag{FP}$$

which is called the fixed point of f. The iteration

$$z_n = f(z_{n-1}) \tag{FPI}$$

for any given $z_0 \in \Omega$ converges to the z satisfying (FP). Now answer the following.

(a) Let $\Omega = [0,1]$ and $f(x) = ax + b$. What are the fixed points for the following cases

$$|a| < 1,\ b \in \mathbb{R},$$

$$|a| = 1,\ b \in \mathbb{R},$$

$$|a| > 1,\ b \in \mathbb{R}.$$

(b) How can one determine L for an f that is continuously differentiable. Given a $z_0 \in \Omega$ consider (FPI) for $j \geq 1$ and set $e_j = z_j - z_{j-1}$. Show that

$$|e_n| \leq L^{n-1}\, e_1.$$

(c) Consider the problem of finding an $x \in [0,1]$ satisfying

$$ax^3 = (1+x)^2$$

for $a > 0$ using (FPI). We have two choices for (FP)

$$f(x) = \sqrt{a}\, x^{\frac{3}{2}} - 1$$

or

$$f(x) = a^{-\frac{1}{3}}(1+x)^{\frac{2}{3}}.$$

For $a = 8$ which one would be your first choice and why? [15]